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THREE DIMENSIONAL FINITE ELEMENT ANALYSIS OF N-PORT WAVEGUIDE JUNCTIONS USING EDGE-ELEMENTS

M. F. WONG, O. PICON and V. FOUAD HANNA

Centre National d'Etudes des Télécommunications
Satellites Telecommunications Systems Division, F92131 Issy-les-Moulineaux, France

ABSTRACT

The finite element method is formulated in such a way that the electromagnetic solution of an excited cavity formed from an N-port waveguides junction leads directly and naturally to circuit characteristics of this junction. The use of edge-elements eliminates non physical solutions. The reliability of the solution is assured compared to a penalty method. Accuracy of the method is demonstrated through the presentation of results for a waveguide T-junction while its efficiency is proven through the presentation of results for a case study of a finline step discontinuity.

II - INTRODUCTION

We need not to recall the utility, in many microwaves applications, of a general method capable to analyse three dimensional waveguide junctions which are inhomogeneously loaded and may include planar circuit configurations. Among possible methods, the finite element seems to be suitable for this purpose. However, the well-known appearance of non physical solutions in either eigenvalue or deterministic problem has limited its use. Some suggestions for eliminating these solutions have been reported [1-3]. The penalty function method [3] for example enforces the divergence free condition to be satisfied, neverthless the efficiency of the method is related to the determination of an appropriate coefficient which is a function of all geometrical and physical problem parameters. This leads to a drastic computational time for the problem solution.

In this paper we propose to eliminate these "spurious modes" by using egde-elements belonging to new finite elements. Standard derivation of Maxwell curl equations is done with special treatment of problem boundary conditions at the ports. It will be shown that this treatment leads to the direct determination of the general impedance or admittance matrices for the N-port waveguide junction between any desired reference planes.

III - EDGE-ELEMENT

Unlike conventional nodal elements, appropriate finite elements [4-10] eliminate the problem of the spurious modes that pollute the frequency spectrum and hence appear also in deterministic problems [10]. For this purpose, a family of these finite elements in tetrahedral and hexahedral mesh, which are conforming in the space H(curl), have been proposed [4]. These elements impose tangential continuity across the elements' boundaries as physically required when they are used to approximate E or H. Since tangential values are the only problem unknowns then boundary conditions are easily prescribed on arbitrary oriented surfaces. Furthermore infinite values of fields need not to be described on metal wedges. In this paper, we use Whitney 1-form edge-element as that proposed by Bossavit in [5,6].

The volume will be meshed by tetrahedra. In each tetrahedron, we define the basis function corresponding to edge e={i,j} as

$$w_e = \lambda_i \nabla \lambda_j - \lambda_j \nabla \lambda_i \qquad (1)$$

where i and j are vertices defining edge e while λ_i and λ_j are barycentric functions associated to nodes i and j (Fig.1).

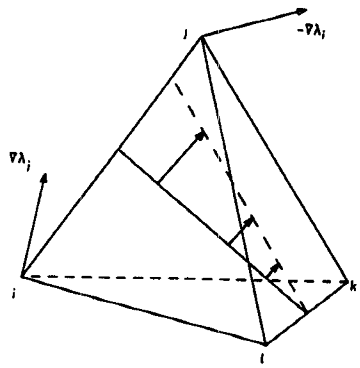

Fig.1 : An edge-element

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IV - FINITE ELEMENT FORMULATION AND IMPEDANCE OR ADMITTANCE MATRICES

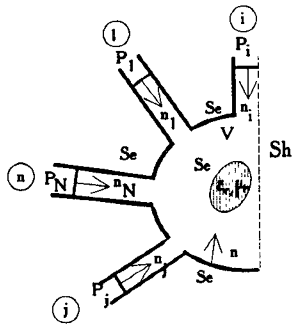

Fig.2 : Typical configuration of an N-port

An N-port waveguide junction (Fig.2) can be defined as a closed volume V containing inhomogeneous materials bounded by an electric wall of surface S_e, a magnetic wall of surface S_h and ports P_i, i=1,N, at reference planes on the junction waveguide arms.

If we assume that each plane P_i is sufficiently far from the junction such that only fundamental mode can propagate in the ith waveguide at this position, one can write

$$n_i \times E = n_i \times V_i e^T_{P_i} = n_i \times (a_i + b_i) e^T_{P_i} \quad \text{on} \quad P_i \tag{4}$$

$$n_i \times H = n_i \times I_i h^T_{P_i} = n_i \times (a_i - b_i) h^T_{P_i} \quad \text{on} \quad P_i \tag{5}$$

where $e^T_{P_i}$ and $h^T_{P_i}$ are normalized transverse fields of the fundamental mode in the ith waveguide while V_i and I_i are reduced voltages and currents corresponding to incident and reflected wave amplitudes, a_i and b_i respectively.

The variational procedure of weighted residuals is applied on Maxwell curl equations and using Green's theorem, the following curl curl equation is obtained :

$$\int_V \frac{1}{\mu_r} \text{curl} E . \text{curl} E' - k^2 \int_V \epsilon_r E . E' \tag{6}$$

$$= - j\omega\mu_0 \sum_i I_i \int_{P_i} \left(n_i \times h^T_{P_i}\right) . E'$$

Using edge-element as basis functions Galerkin's method leads to the following matricial equations :

$$\overline{E} = - j\omega\mu_0 \sum_i I_i (\overline{A} - k^2 \overline{B})^{-1} \overline{J}_{P_i} \tag{7}$$

where

$$(\overline{A})_{e,e'} = \int_V \frac{1}{\mu_r} \text{curl} w_e . \text{curl} w_{e'} \tag{8}$$

$$(\overline{B})_{e,e'} = \int_V \epsilon_r w_e . w_{e'} \tag{9}$$

$$\left(\overline{J}_{P_i}\right)_e = \int_{P_i} \left(n_i \times h^T_{P_i}\right) . w_{e,P_i} \quad \text{if} \quad e \in P_i \tag{10}$$

$$= 0 \quad \text{elsewhere}$$

Equation (7) is equivalent to the solution of N matrix equations with the same first member matrix $(\overline{A} - k^2 \overline{B})$ and N different second member vectors $\overline{J}_{P_i}$. The corresponding solutions are $(\overline{A} - k^2 \overline{B})^{-1} \overline{J}_{P_i} = \overline{e}_i$. $\overline{e}_i$ is the field configuration when all ports are open-ended except port i in which $h^T_{P_i}$ is applied.

If e_i is the edge-element approximation associated to $\overline{e}_i$

$$E = - j\omega\mu_0 \sum_i I_i e_i \tag{11}$$

multiplying by $n_j \times$ on port j, using (4), multiplying by $h^T_{P_i}$ and integrating on P_j, one obtains :

$$V_j = - j\omega\mu_0 \sum_i \int_{P_i} (n_j \times e_i) . h^T_{P_i} I_i \tag{12}$$

The last relation can be expressed as :

$$\overline{V} = \overline{Z} \overline{I} \tag{13}$$

and

$$[Z]_{j,i} = j\omega\mu_0 \overline{J}^t_{P_j} \overline{e}_i \tag{14}$$

$$= j\omega\mu_0 \overline{J}^t_{P_j} (\overline{A} - k^2 \overline{B})^{-1} \overline{J}_{P_i} \quad i, j = 1 \ \ to \ \ N$$

Each solution e_i allows the calculation of one column of impedance matrix $\overline{Z}$. The dual formulation with H can also be derived in the same manner and an admittance matrix is obtained.

V - NUMERICAL RESULTS AND COMPARISONS

To show the importance of the chosen edge-elements for the proper description of fields inside a waveguide junction, an E-plane waveguide T-junction in the Ka band (Fig.3) is studied. Of course, the excitation field patterns are those for the fundamental TE_{01} mode. Fig.4 gives the electric field distribution in the T-junction using

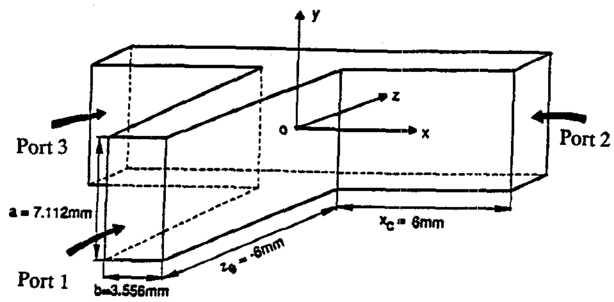

Fig.3 : E-plane T-junction

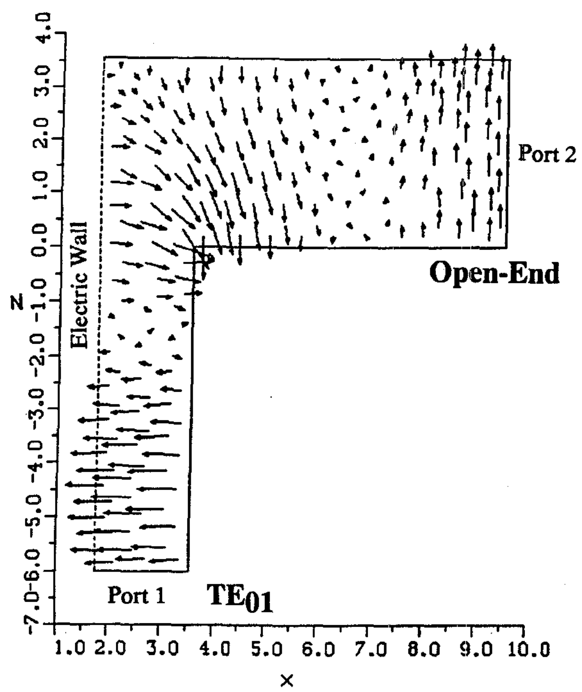

Fig.4 : Electric field pattern in the y=0 plane when port 2 and 3 are open-ended and port 1 excited for a T-junction using edge elements

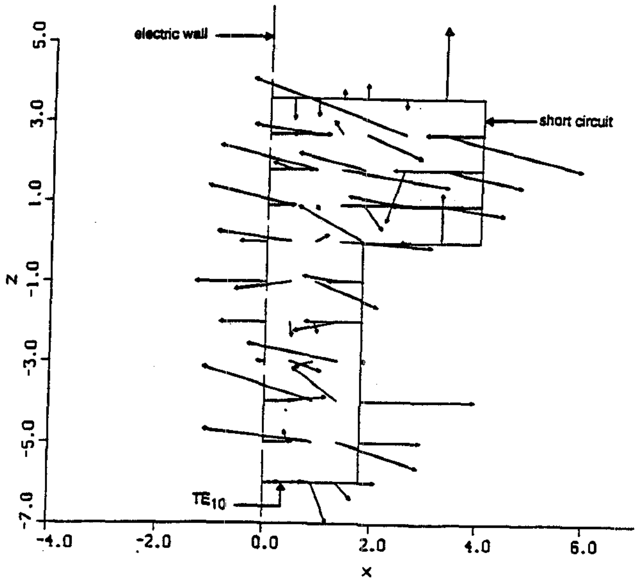

Fig.5 : Electric field pattern in the y=0 plane when port 2 and 3 are short-circuited and port 1 excited for a T-junction using nodal elements

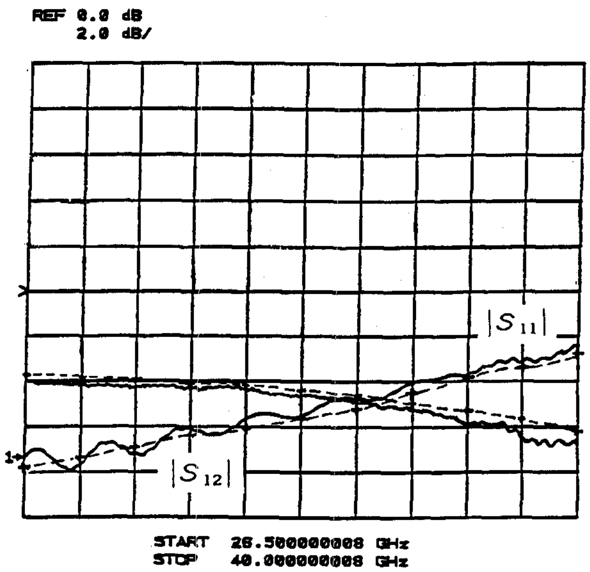

Fig.6 : Experimental (solid lines) and numerical (dashed lines) results for an E-plane T-junction : $|S_{11}|$ *and* $|S_{12}|$

edge elements, while Fig.5 gives results for the same problem when using conventional nodal elements. Non physical solutions are detected in the second case while proper amplitudes and orientation of field pattern and consequently correct values for the junction S-parameters are obtained in the first case. For nodal elements a penalty function [3] has to be used to get correct field pattern, while to get precise values for the junction S-parameters an appropriate multiplying factor to the function must be chosen [3]. Then the 3-port T-junction is analysed. The computed S-parameters are then compared to experimental results (Fig.6). Using about 16,000 unknowns mesh, a good agreement between theory and experiment is obtained.

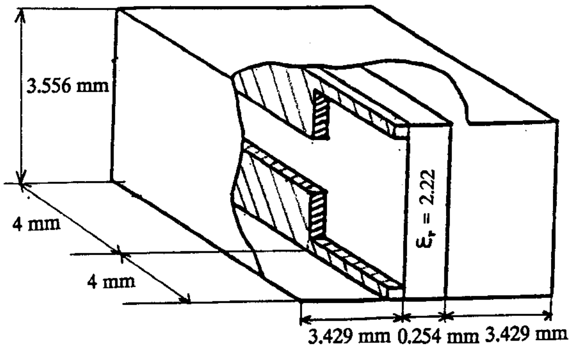

Fig.7 : Finline step discontinuity

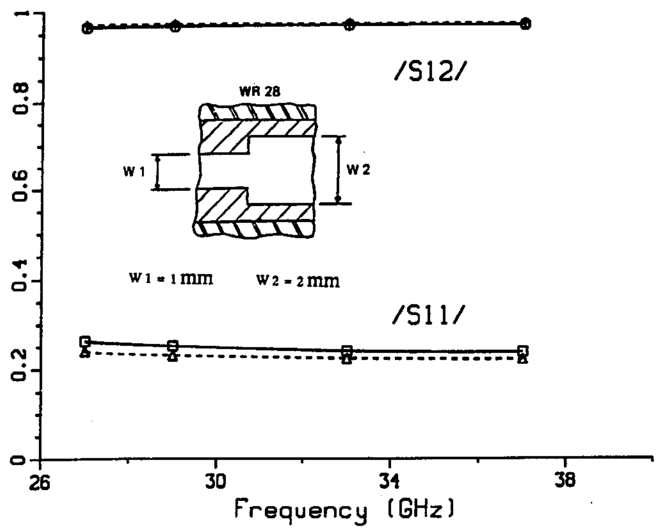

Fig.8 : Numerical results for $|S_{11}|$ and $|S_{12}|$ for a finline step discontinuity : spectral domain analysis (dashed lines) and finite element analysis (solid lines)

Finally, a finline step discontinuity (Fig.7) is meshed giving about 10,000 unknowns. The results are compared to those obtained using a spectral domain approach combined with modal analysis [13]. An example of results is given in Fig.8. The relative error is less than 1% for S_{12}.

VI - REFERENCES

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